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Technical Report 32-1603

JPL Development Ephemeris Number 96

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

February 29, 1976

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JPL Development Ephemeris Number 96

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

February 29, 1976

Preface

The work described in this report was performed by the Mission Analysis Division of the Jet Propulsion Laboratory.

Acknowledgment

The program for producing ephemerides at JPL has been in existence for many years. Therefore, it is not possible to acknowledge the many people who have contributed to the long succession of improvements. However, the authors wish to express their gratitude to a few people who have been most closely involved.

In the past, Dr. Jay H. Lieske was in charge of the ephemeris group. He has continued to show an interest in the ephemeris effort and has contributed many valuable discussions.

Similarly, Dr. James G. Williams, who is developing the lunar ephemerides, has lent much appreciated advice to our effort. He and W. S. Sinclair provided us with the lunar ephemeris.

At present, Dr. John D. Anderson directs the planetary ephemeris development. His advice and encouragement are greatly appreciated.

We owe our gratitude to those who have generously supplied us with observational data: to Dr. P. K Seidelmann (USNO), Dr. C. Oesterwinter (NWL), and Dr. D. A. O'Handley (JPL) for the optical data; Drs. G. H. Pettengill and I. I. Shapiro (MIT) and Drs. R. M. Goldstein, G. S. Downs, and P. E. Reichley (JPL) for the radar data; Dr. J. F. Jordan (JPL) for the Mariner 9 data; and D. L. Cain, A. Liu, and G. W. Null (JPL) for the Pioneer 10 and 11 data.

Finally, we express our gratitude to Mrs. Eunice L. Lau, who processed the Mariner 9 data for our use.

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Abstract

The fourth issue of JPL Planetary Ephemerides, designated JPL Development Ephemeris No. 96 (DE96), is described. This ephemeris replaces DE69 (Ref. 1), which has become obsolete since its release in 1969.

The improvements in DE96 are many. They include more recent and more accurate observational data, new types of data, better processing of the data, and refined equations of motion which more accurately describe the actual physics of the solar system. The descriptions in this report include those new features as well as the new export version of the ephemeris.

The tapes and requisite software will be distributed through the NASA Computer Software Management and Information Center (COSMIC) at the University of Georgia.

JPL Development Ephemeris Number 96

I. Introduction

The JPL Development Ephemeris 96 described in this report is the fourth official release from the JPL Ephemeris Tape System. It replaces DE69, which was released in 1969 (Ref. 1) and which has since become obsolete.

The JPL Ephemerides are produced by a system of programs referred to as the Solar System Data Processing System (SSDPS). Observational data are collected and compared against a base ephemeris. Partial derivatives are computed and used in a differential least-squares program to improve the values of the initial conditions of the planets at a given epoch, along with other associated parameters. The differential equations of motion, using the new initial conditions, are numerically integrated, thus producing the new ephemeris. Finally, the observational data are compared against the new ephemeris as a check.

This report discusses the main features that were present in the production of DE96. The following sections discuss the observations and how they are processed, the formation of the partial derivatives, the solution and its resultant constants and initial conditions, the numerical integration and equations of motion, the final residuals, and, lastly, the export version of DE96.

II. The Observational Data

There were 44002 observations used in the solution for DE96. These come from five major sources, which are described below. Different weights were assigned to various sets of observations according to the formula, $\sqrt{w} = 1/\sigma_0$, where σ_0 is the a priori standard deviation. From previous experience, we were able to assign values to these which were approximately equal to the post-fit rms residuals.

A. Optical

The optical observations come from the Six- at A Nine-Inch Transit Circles of the U. S. Naval Observatory (USNO). They cover the time span, 1911–1971 and have been discussed by Oesterwinter and Cohen (Ref. 2) and by O'Handley et al. (Ref. 1).

All of the optical observations have been reduced to the FK4 Catalogue system using the tables given in the Second Series of the *Publications of the U. S. Naval* Observatory.

The a priori standard deviations in right ascension and declination were 1".0 sec δ and 1".0, respectively, for the Sun, Mercury, and Venus and 0".5 sec δ and 0".5, respectively, for Mars through Neptune.

The number of observations for each body and the rms post-fit residuals for the optical data are shown in Table 1.

B. Radar

Radar time-delay measurements from Mercury, Venus, and Mars have come from six sources: Arecibo Ionospheric Observatory, Haystack (MIT), Millstone Hill (MIT), Goldstone Deep Space Station (DSS) 13 (JPL), Goldstone DSS 13/DSS 14 Bistatic (JPL), and Goldstone DSS 14 (JPL). The a priori standard deviations assigned to these data varied according to source, planet, and year. The number of radar data points for each planet and their post-fit rms residuals are shown in Table 2.

C. Mariner 9 Range Points

The Mariner 9 Navigation Team combined 804 range points to the Mars Orbiter with positions of the orbiter to the center of mass of Mars in order to produce accurate Earth–Mars ranges from November 1971 to October 1972. These data, shown in Table 3, exist in four sets according to their proximity to the Martian solar conjunction (JD 2441568), when the 2200-MHz ranging signal passed within 4 solar radii of the Sun at the heliographic latitude of +79°. The uncertainties in the propagation of the signal through the corona are reflected by the post-fit rms residuals. The modeling of the corona is discussed in Section IIID.

D. Pioneer 10 and 11

The Pioneer Navigation Teams provided Earth–Jupiter ranges by combining Earth–spacecraft ranges with positions of the spacecraft relative to Jupiter's center of mass at the times of encounter (JD 2442020 and JD 2442385). The a priori standard deviations were 50 μs for each.

E. 1971/1973 Mars Radar Closure

There were 291 "closure points" from the Mars radar data taken during the oppositions of Mars in 1971 and in 1973. These closure points are pairs of days during which the observed points on the surface of Mars are nearly identical with respect to Martian longitude and latitude. Since the same topographical features are observed during each day, the uncertainty introduced by the topography of Mars may be eliminated by subtracting the residuals of one day from those of the other day. The remaining difference is then due only to the ephemeris drift between the two days. These points had a priori standard deviations of about 1 µs.

III. Processing of Observations

In addition to the standard calculations used when comparing the observations against corresponding values predicted by the ephemeris, there were some unique features in DE96, which are described below.

A. Day Corrections

The optical data from the USNO covering the years 1962–1971 have not yet had day corrections applied for the Sun, Mercury, and Venus; i.e., the bodies observed during daylight hours. Consequently, temporary corrections are applied in DE96, the coefficients being determined in the solution itself. The forms of the corrections are

$$\Delta \alpha = A_1 + A_2 \sin \delta + A_3 \cos h_0$$

$$\Delta \delta = D_1 + D_2 \sin \delta + D_3 \cos h_0$$

where δ is the declination and h_{\odot} is the hour angle of the Sun (i.e., time of day). The corrections, $\Delta \alpha$ and $\Delta \delta$, are to be added to the *computed* values of α and δ .

B. Corrections to Drift in the Optical Data

In recent years, it has become apparent that the residuals for the inner planets (i.e., those observed by radar) have shown a secular-like drift in right ascension similar to what would be shown by inaccurate precession and equinox drift. As such, the corrections $\Delta \alpha = (\Delta m + \Delta n)$

sin α tan δ)T and $\Delta\delta$ = $(\Delta n \cos \alpha)T$, similar to precession formulae (see, e.g., Ref. 12, paragraph 137), and $\Delta\alpha$ = (E)T, similar to equinox drift, are combined into

$$\Delta \alpha = (\Delta k + \Delta n \sin \alpha \tan \delta) T$$

and

$$\Delta \delta = (\Delta n \cos \alpha) T$$

where E is the rate of drift and T is measured in centuries past 1950.0. The coefficients Δk and Δn are determined in the solution; the corrections $\Delta \alpha$ and $\Delta \delta$ are to be *subtracted* from the *observed* values of α and δ . These values are *not* incorporated into the precession parameters; they serve only as modifications to the optical data.

C. Corrections to Limb Biases for Mercury and Venus

Most of the transit observations of Venus and some of Mercury are taken not on the center of light but rather on the illuminated edge. The corrections to center of planet are then applied by the USNO, using values of parameters currently available. To account for corrections to these values, the following formulae are applied:

$$\Delta \alpha = \frac{\pm r_a}{\rho}$$
 and $\Delta \delta = \frac{\pm r_b}{\rho}$

where the corrections r_a and r_b are determined in the solution, ρ is the Earth-planet distance, and the sign-depend on which limb is being observed.

D. Solar Corona Time Delay

Besides the standard relativistic time delay in the radar signals between the Earth and a planet (Ref. 4), there is also a delay caused by the electron density in the solar corona. This has been discussed by Muhleman et al. (Ref. 5). The following formula for corona delay $\Delta \tau$ (μ s) was used in processing the radar and Mariner 9 data in DE96:

$$\Delta \tau = \frac{40.3}{cf^2} \int_{P_1}^{P_2} N_e \, ds$$

where c is the speed of light (cm/s), f is the frequency (MHz) of the radio carrier signal, N_e is the electron density (cm⁻³), and the integration is carried out over the linear distance (cm) from point P_1 to point P_2 in space. The electron density was assumed to have the following form:

$$N_e = \frac{A}{r^0} + \frac{B}{r^{2-\epsilon}}$$

with the solar distance r expressed in units of the solar radius.

The values of the arbitrary constants used for DE69 were

$$A = 1.13 \times 10^{8} \text{ cm}^{-3}$$

 $B = 0.5 \times 10^{6} \text{ cm}^{-3}$
 $r = 0.0$

These values are consistent with the corona derived from Mariner 6 and 7 data by Muhleman et al. (Ref. 5).

E. Mars Ellipsoidal Model

Occultation measurements of the Martian surface by Mariner 9 have shown that the shape of the surface may be approximated by a triaxial ellipsoid. This determination and its implications for radar ranging have been discussed by Standish (Ref. 6). The radar time delays from Mars have been computed using this model. The whole ellipsoid is scaled according to the mean equatorial radius of the planet, the only associated parameter in the solution for DE96. The shape and orientation of the ellipsoid are unaltered.

The surfaces of Mercury and Venus are approximated by spheres.

IV. Partial Derivatives

The partial derivatives used in the observation equations were obtained, for the most part, from the Set III formulation of Brouwer and Clemence (Ref. 7, p. 241). For a given observation α , say, this calculation invokes the following chain rule:

$$\frac{\partial \alpha(t)}{\partial S(0)} = \frac{\partial \alpha(t)}{\partial \mathbf{r}(t)} \frac{\partial \mathbf{r}(t)}{\partial S(0)}$$

where $\mathbf{r}(t)$ is the vector of Cartesian coordinates at time t, and $\mathbf{S}(0)$ is the vector of Set III corrections at epoch (t=0). The second factor on the right-hand side is the matrix given by Brouwer and Clemence where osculating elements are typically used. It is rigorously exact for only true Keplerian motion or at epoch. In the actual cases, the accuracy was found to be good enough to support nearly all of the observations. The exceptions are described below:

(1) The heliocentric orbit of Neptune is poorly approximated by Keplerian motion. A dramatic improvement is seen when the motion is with respect to the barycenter of all bodies interior to Neptune. Therefore, in the case of Neptune, the formula above was replaced by

$$\frac{\partial \alpha(t)}{\partial S(0)} = \frac{\partial \alpha(t)}{\partial \mathbf{r}^*(t)} \frac{\partial \mathbf{r}^*(t)}{\partial S^*(0)} \frac{\partial S^*(0)}{\partial S(0)}$$

where the starred quantities are computed with respect to the barycenter interior to Neptune. Here, the second factor is far more accurate than its unstarred counterpart above. The mird factor needs to be computed only once. It comes from

$$\frac{\partial S^{\star}(0)}{\partial S(0)} = \frac{\partial S^{\star}(0)}{\partial \mathbf{r}^{\star}(0)} \frac{\partial \mathbf{r}^{\star}(0)}{\partial \mathbf{r}(0)} \left[\frac{\partial S(0)}{\partial \mathbf{r}(0)} \right]^{-1}$$

where one may see that

$$\frac{\partial \mathbf{r}^{\star}(0)}{\partial \mathbf{r}(0)} = \mathbf{I}$$

- (2) The high precision of the Mariner 9 range data requires exceptionally accurate partials. For these, numerically integrated variational equations including the effect of Jupiter's orbit were used.
- (3) The 1971–1973 Mars radar closure analysis also has an inherently high degree of precision. Numerically obtained partials were used here as well.

V. Solution for DE96

There were 64 parameters in the solution for DE96. A full rank solution of an eigenvalue-eigenvector analysis was applied to the osculating elements at epoch JD = 2440400.5. The parameters were

- 48 orbital elements (Pluto excluded)
- 4 limb corrections for Mercury and Venus

- 3 radii for Mercury, Venus, and Mars
- 1 scale factor (km/AU)
- 6 transit circle day corrections for the Sun, Mercury, and Venus covering 1962–1971
- 2 corrections for the drift in the optical data

Tables 4, 5, and 6 give heliocentric 1950.0 equatorial initial conditions for all nine planets plus the Moon for three different epochs. Tables 7 and 8 present values of the other parameters used in DE96.

We have adopted the set of planetary masses which are being recommended to the International Astronomical Union (IAU) by Commission 4, with the exception of the mass of the Earth-Moon barycenter, which has one more digit in DE96.

For the orbit of Pluto, we have used the initial conditions from DE69.

VI. Covariance/Correlation Matrix of DE96

The formal standard deviations and the correlation matrix from the solution for DE96 are given in Table 9. The units are arc seconds for all parameters except for the astronomical unit and the three radii, which are in kilometers.

The following list provides identification of the computerized version of the parameter names:

DMWi	$\Delta \ell + \Delta r$	Brouwer and Clemence Set III
DPi	Δp	elements for ith planet.
DQi	Δq	
EDWi	$e\Delta r$	
DAi	$\Delta a/a$	
$\mathrm{DE}i$	Δe	
AU		Scale factor (km/AU)
RRAi	r_a	Limb corrections for ith planet
RDEi	r_{δ}	
ADAY1	A_1	Day corrections
DDAY3	D_3	

DELK
$$\Delta k$$
 Optical data drift corrections
DELN Δn

It must be emphasized that Table 9 gives formal values obtained directly from the solution. It is well known that the use of such formal covariances often leads to overly optimistic predictions of accuracy.

VII. Numerical Integration of the Planets

The dynamic evolution of the solar system was obtained by numerically integrating the equations of motion over the entire twentieth century. The gravity model used is the isotropic, Parameterized Post-Newtonian (PPN) n-body metric (Ref. 8) and the Newtonian gravity perturbations of the asteroids Ceres, Pallas, and Vesta regarded as following heliocentric Keplerian ellipses. The celestial bodies being integrated are the Sun and the nine planets. The geocentric lunar ephemeris LE-44 was obtained by an independent integration and was treated as input by the planetary program.

The *n*-body equations of motion were derived from the variation of a time-independent Lagrangian action integral formulated in a nonrotating solar-system barycentric Cartesian coordinate frame. For each celestial body, the *n*-body equations of motion are, to order $1/c^2$,

$$\begin{split} \ddot{\mathbf{r}}_{i} &= \sum_{j \neq i} \frac{\mu_{j} \left(\mathbf{r}_{j} - \mathbf{r}_{i}\right)}{r_{ij}^{3}} \left\{ 1 - \frac{2 \left(\beta + \gamma\right)}{c^{2}} \sum_{k \neq i} \frac{\mu_{k}}{r_{ik}} \right. \\ &- \frac{2\beta - 1}{c^{2}} \sum_{k \neq j} \frac{\mu_{k}}{r_{jk}} + \gamma \left(\frac{\upsilon_{i}}{c}\right)^{2} + \left(1 + \gamma\right) \left(\frac{\upsilon_{j}}{c}\right)^{2} \\ &- \frac{2 \left(1 + \gamma\right)}{c^{2}} \dot{\mathbf{r}}_{i} \cdot \dot{\mathbf{r}}_{j} - \frac{3}{2c^{2}} \left[\frac{\left(\mathbf{r}_{i} - \mathbf{r}_{j}\right) \cdot \dot{\mathbf{r}}_{j}}{r_{ij}} \right]^{2} \\ &+ \frac{1}{2c^{2}} \left(\mathbf{r}_{j} - \mathbf{r}_{i}\right) \cdot \ddot{\mathbf{r}}_{j} \right\} + \frac{1}{c_{2}} \sum_{j \neq i} \frac{\mu_{j}}{r_{ij}^{3}} \left\{ \left[\mathbf{r}_{i} - \mathbf{r}_{j}\right] \right. \\ &\cdot \left[\left(2 + 2\gamma\right) \dot{\mathbf{r}}_{i} - \left(1 + 2\gamma\right) \dot{\mathbf{r}}_{j} \right] \right\} \left(\dot{\mathbf{r}}_{i} - \dot{\mathbf{r}}_{j}\right) \end{split}$$

$$+ \frac{3+4\gamma}{2c_z} \sum_{j\neq i} \frac{\mu_j \ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m} \frac{\mu_m \left(\mathbf{r}_m - \mathbf{r}_i\right)}{r_{im}^z}$$

+
$$\sum_{j\neq i} 3 J_{2j}\mu_j \frac{a_j^2}{r_{ij}^4} \left[\left(\frac{5}{2} \left(\frac{\mathbf{r}_i - \mathbf{r}_j}{r_{ij}} \cdot \mathbf{p}_j \right)^2 \right) \right]$$

$$-\frac{1}{2}\left(\frac{(\mathbf{r}_i - \mathbf{r}_f)}{r_{ij}} - \left(\frac{(\mathbf{r}_i - \mathbf{r}_f)}{r_{ij}} \cdot \mathbf{p}_i\right) \mathbf{p}_j\right]$$
(1)

where

 \mathbf{r}_i , $\dot{\mathbf{r}}_i$, and $\ddot{\mathbf{r}}_i$ are the barycentric position, velocity, and acceleration vectors of body i

 $m_i = Gm_j$, where G is the gravitational constant and m_j is the mass of body j

$$r_{ij} = |\mathbf{r} - \mathbf{r}_i|$$

 β is the PPN parameter measuring the nonlinearity in superposition of gravity

 γ is the PPN parameter measuring space curvature produced by unit rest mass (In this integration, as in general relativity, $\beta = \gamma = 1$.)

$$v_i = |\dot{\mathbf{r}}_i|$$

c is the velocity of light

 J_{zj} is the second zonal harmonic coefficient of body j; a_j is the equatorial radius of body j

 \mathbf{p}_{j} is the unit vector in the direction of the north celestial pole of body j

The index m in the next-to-last term on the right side of (1) runs over the asteroids. The quantity $\ddot{\mathbf{r}}_j$ appearing in two terms on the right side of (1) includes the Newtonian acceleration of each body j due to all other major celestial bodies and the three asteroids:

$$\ddot{\mathbf{r}}_{j} = \sum_{k \neq j} \frac{\mu_{k} \left(\mathbf{r}_{k} - \mathbf{r}_{j}\right)}{r_{jk}^{3}}$$

where j runs over the Sun and planets and k runs over the Sun, planets, and asteroids. The last term on the right side of (1) represents the contribution of the oblateness effects of each body. For this integration, all harmonic coefficients J_{2j} were set to zero. Normally, the only significant contribution arises from the oblateness of the Sun.

The independent variable of integration is the time coordinate in the n-body metric.

The masses and Keplerian elements of the asteroids are given in Table 10.

The definition of the relativistic barycenter (center of mass) of the solar system is somewhat modified from the usual Newtonian formulation (Ref. 9). From conservation of linear and angular momentum, the location of the barycentric origin is given by

$$\sum_{\ell} \mu_j^* \mathbf{r}_j = 0 \qquad (2)$$

where

$$\mu_{j}^{*} = \mu_{j} \left\{ 1 + \frac{1}{2c^{2}} v_{j}^{2} - \frac{1}{2c^{2}} \sum_{k \neq j} \frac{\mu_{k}}{r_{jk}} \right\}$$
(3)

Only the planets were actually integrated. At each step throughout the integration the relativistic masses μ_j^* were calculated from (3), and the position of the Sun was subsequently determined from (2). Because of substantial uncertainty μ_j eir masses, the asteroids were not included in (1, 1) centric calculations.

VIII. Lunar Ephemeris

The construction of LE44 is similar to that of the LURE2 ephemeris, which is to be documented elsewhere.

Because there was no attempt to put the lunar and the planetary ephemerides on the same reference system, there may be rotations in all three axes amounting to no more than 0.1 arc second between the two. The improved masses of DE96 were used and the secular acceleration of the Moon's longitude was adopted, without fitting, to be $\dot{n} = -38"/\text{century}^2$.

IX. Residuals With Respect to DE96

The residuals for the observational data with respect to DE96 are shown in Figs. 1–4. The post-fit standard deviations have been given in Tables 1–3.

The optical residuals, plotted in Figs. 1a-h, appear similar to those given by Oesterwinter and Cohen (Ref. 2), whose ephemeris was formed from the optical data only.

This seems to indicate that the other data in DE96 have not introduced any major inconsistencies. We have, however, noticed systematic errors in the optical data, as did Oesterwinter and Cohen. These are discussed in the next section.

The radar residuals are plotted in Figs. 2a-c. The early data have been severely down-weighted. There seem to be trends due to topography in the plots of Mercury and Venus, these having been modeled simply as spherical bodies. For Mars, the residuals are with respect to the ellipsoid referred to in Section III. In all three cases, the standard deviations are about $10~\mu s$, amounting to about 1.5~km. This would increase to about 2.2~km for a spherical model of the Martian surface.

The Mariner 9 residuals are shown in Fig. 3. The poor fit at the Martian solar conjunction is apparent. Here the data are not only noisy but corrupted by large fluctuations in the solar corona. Further analysis of these data is still being performed.

The residuals in range for Pioneer 10 and 11 are -47 and $+67 \mu s$, respectively.

The residuals of the Mars closure analysis are shown in Fig. 4 by means of a histogram. Of the 291 residuals, 206 were less than 1 μs . The largest residual was 6.5 μs . Most of the 16 residuals greater than 3 μs come from observations where the altitude on the surface of Mars is rapidly increasing with changing latitude. If the two days being compared have significantly different latitudes (up to 1° was allowed), a 6.5 μs residual can arise from only a 1% incline.

X. A Systematic Trend in the Right-Ascension Residuals

The right-ascension residuals of the planets contain a systematic trend which is not evident in Figs. 1a-n because the plots are so compressed in time. Therefore, the right-ascension residuals of Mercury through Neptune are plotted again in Figs. 5a-g, where this time the abscissa corresponds to the heliocentric difference in right ascension between the Earth and the planet; i.e., degrees past opposition (degrees past inferior conjunction for Mercury and Venus).

The cause of this trend is unknown, but it is definitely a universal one—not due to one particular planet, ephemeris, or set of observations. The reasoning for this is as follows:

- The trend is obvious in Figs. 5a-d for Mercury through Jupiter.
- (2) The trend has been noted in the residuals for the USNO Ephemeris (Ref. 10), in the Dahlgren Ephemeris, where it was actually discussed for Venus by Oesterwinter and Cohen (Ref. 2), and in Laubscher's dissertation for Mars (Ref. 11), as well as in the JPL Ephemerides to date.
- (3) The trend is present in the Tokyo and Greenwich transit data as well as the USNO data, these three sources being present in Laubscher's dissertation.

This systematic trend, the "opposition effect" or "phase effect," is presently being investigated.

XI. Export Ephemeris

The DE96 ephemeris, along with all requisite software, has been put on magnetic tape in a format suitable for a wide range of computers.

The new form t is more than four times as compact as the previous "Type 50" used for DE69. Furthermore, the interpolation error is below 10 cm for all bodies—an improvement of 10 from before.

The range of data is from December 16, 1944, to January 25, 2000.

Copies of the tape, software, and user instructions are available from the NASA Computer Software Management and Information Center (COSMIC) at the University of Georgia.

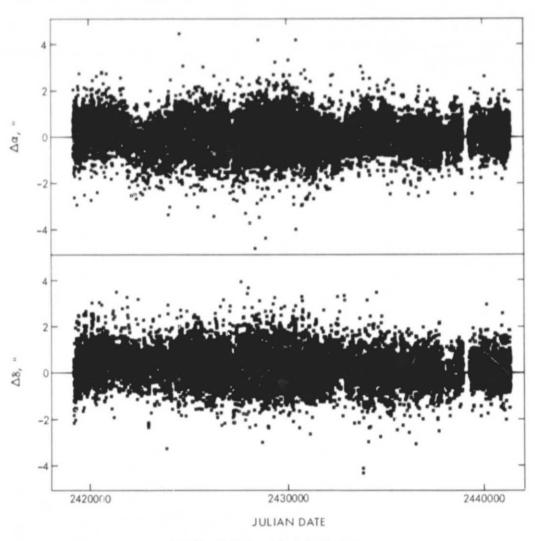


Fig. 1a. Optical residuals for the Sun

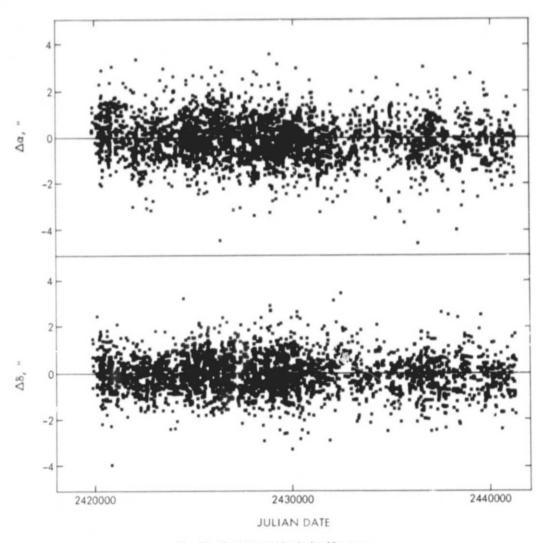


Fig. 1b. Optical residuals for Mercury

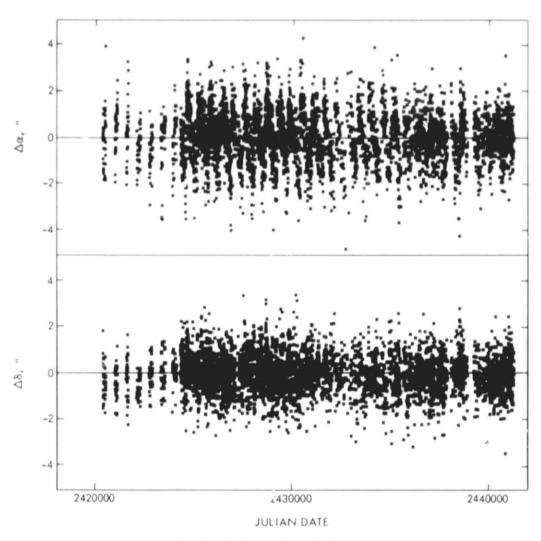


Fig. 1c. Optical residuals for Venus

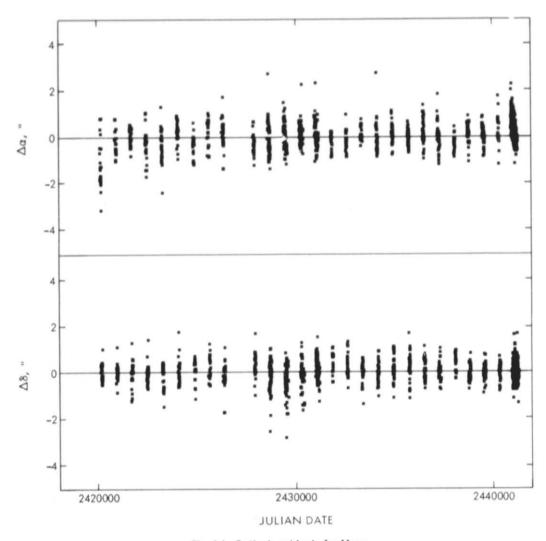


Fig. 1d. Optical residuals for Mars

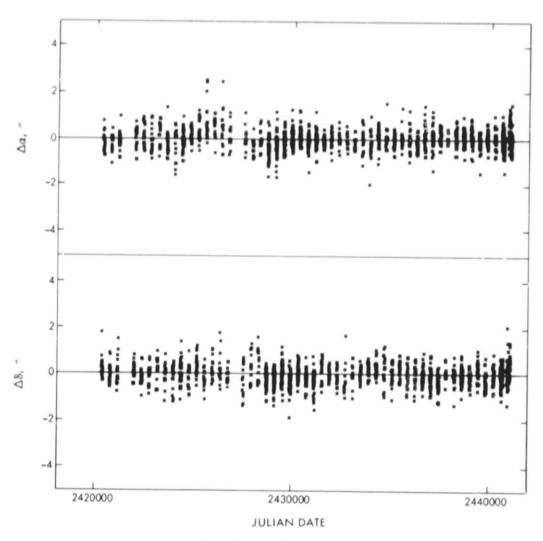


Fig. 1e. Optical residuals for Jupiter

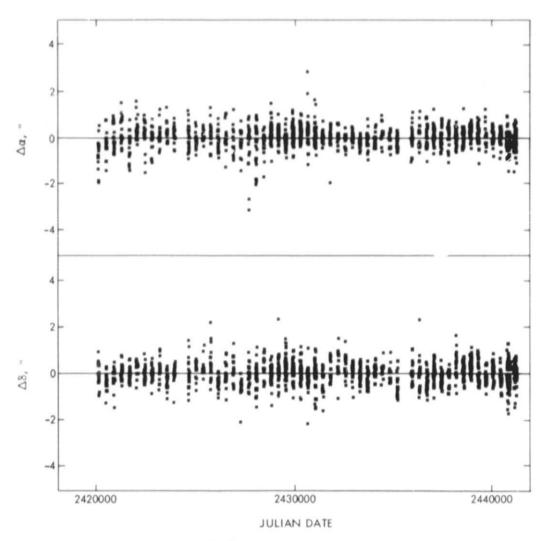


Fig. 1f. Optical residuals for Saturn

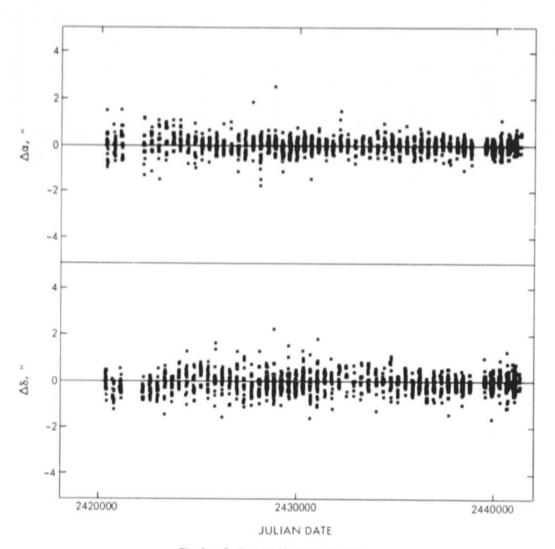


Fig. 1g. Optical residuals for Uranus

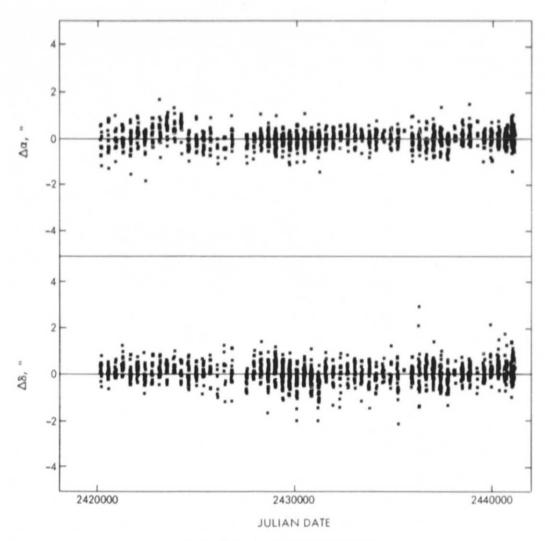


Fig. 1h. Optical residuals for Neptune

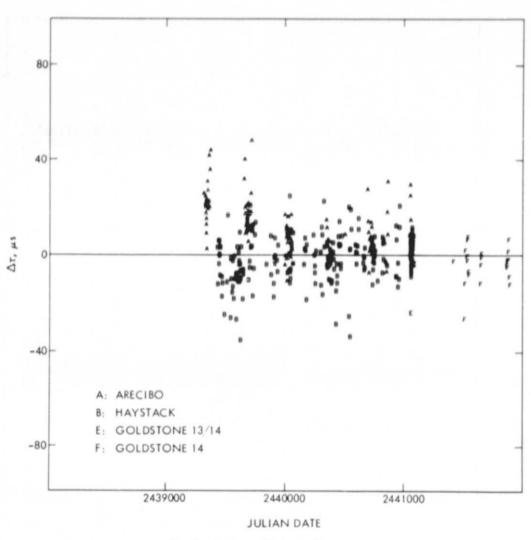


Fig. 2a. Radar residuals for Mercury

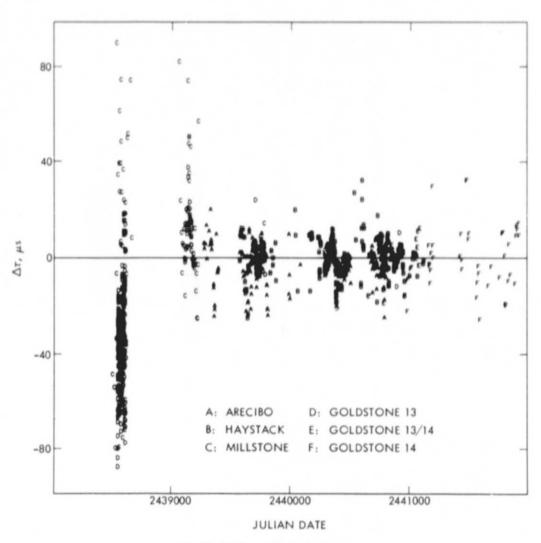


Fig. 2b. Radar residuals for Venus

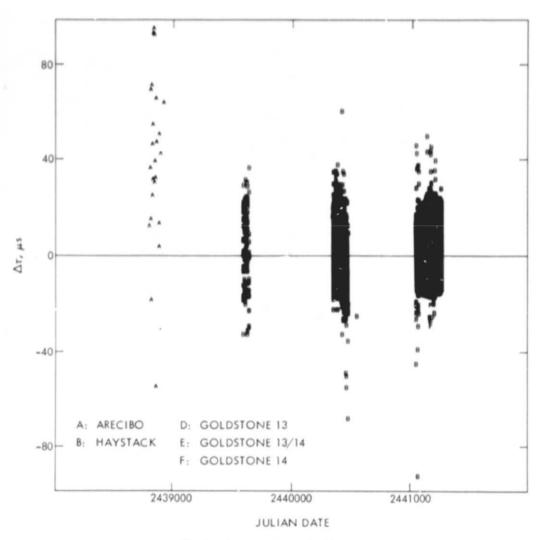


Fig. 2c. Radar residuals for Mars

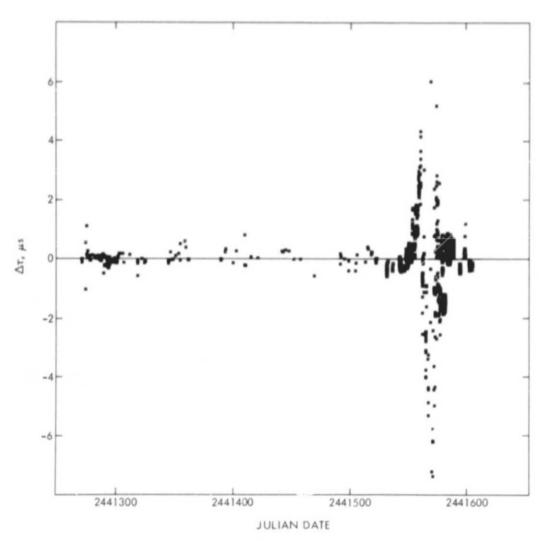


Fig. 3. Mariner 9 range residuals for Mars

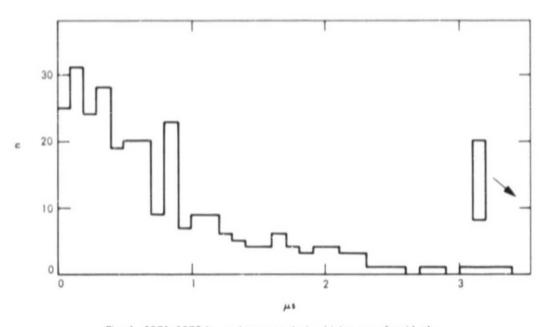


Fig. 4. 1971/1973 N. s closure analysis—histogram of residuals

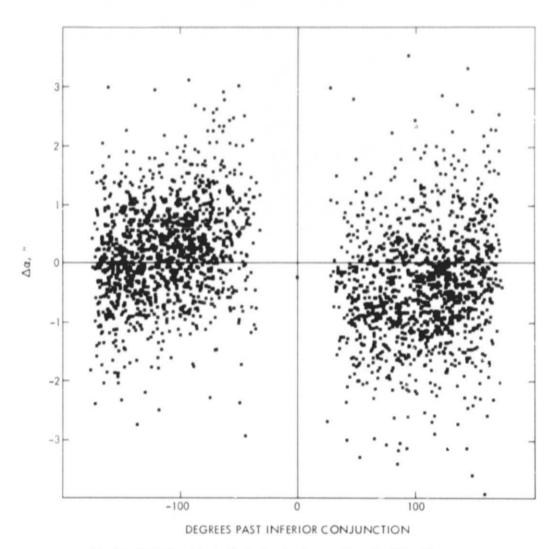


Fig. 5a. Optical residuals illustrating the "opposition effect" for Marcury

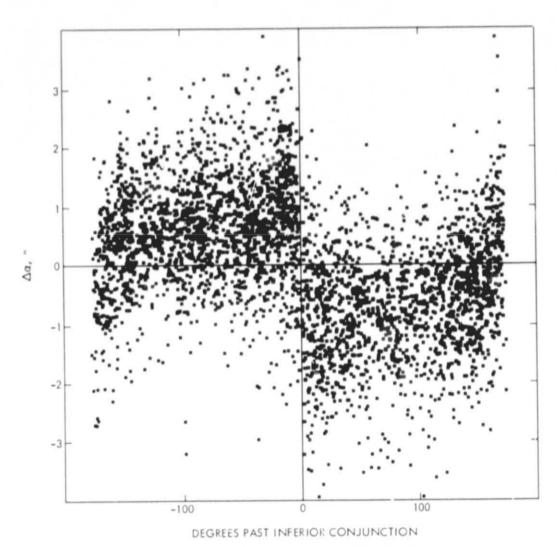


Fig. 5b. Optical residuals illustrating the "opposition effect" for Venus

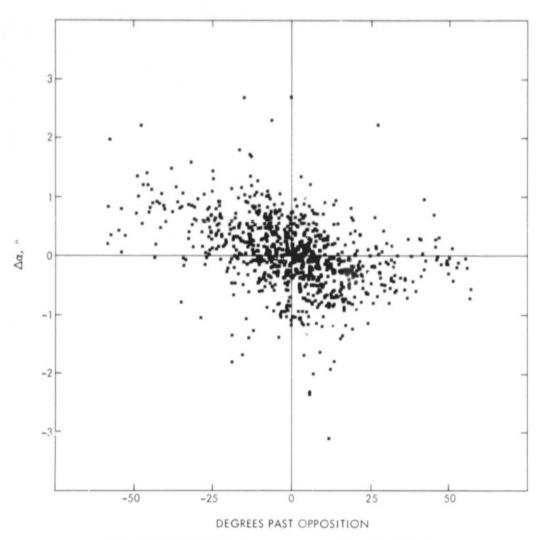


Fig. 5c. Optical residuals illustrating the "opposition effect" for Mars

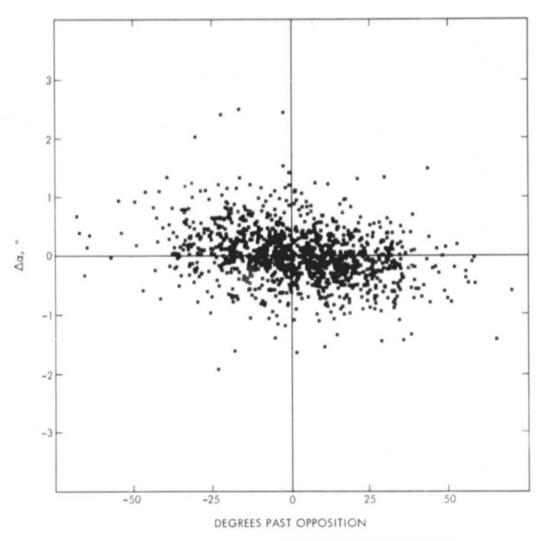


Fig. 5d. Optical residuals illustrating the "opposition effect" for Jupiter

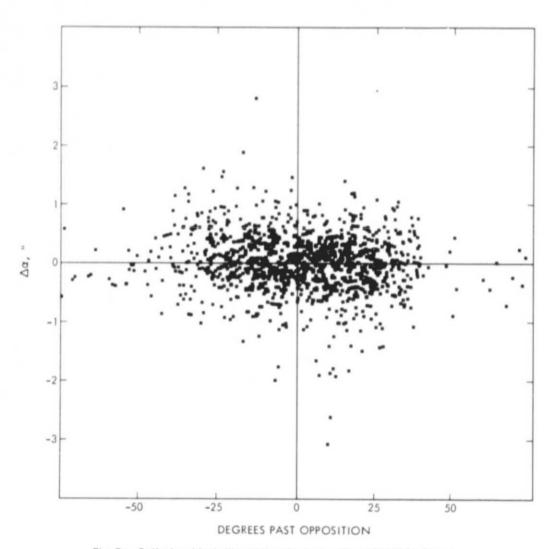


Fig. 5e. Optical residuals illustrating the "opposition effect" for Saturn

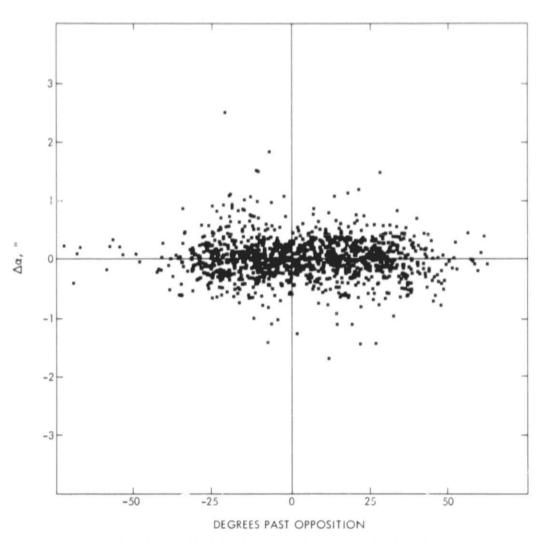


Fig. 5f. Optical residuals illustrating the "opposition effect" for Uranus

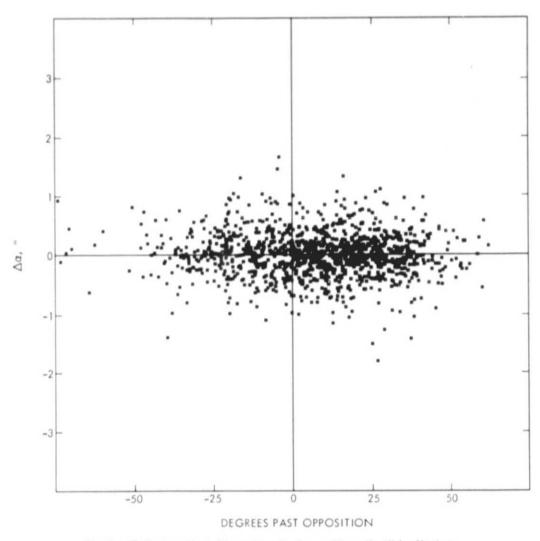


Fig. 5g. Optical residuals illustrating the "opposition effect" for Neptune

Table 1. Optical observations

Planet	N_a	σ_a , "	N_{δ}	σ_{δ} , "
Sun	8223	0.81	7930	0.83
Mercury	2412	0.98	2339	0.85
Venus	3566	1.17	3386	0.89
Mars	330	0.63	804	0.55
Jupiter	1068	0.50	1030	0.51
Saturn	1091	0.53	1040	0.54
Uranus	1048	0.37	1034	0.45
Neptune	1037	0.40	1015	0.50

Table 2. Radar observations

£	Mer	cury	Ve	nus	s Ma		
Source N		σ, μs	N	σ, μs	N	σ, μs	
Arecibo	106	17.1	248	8.1	30	83.3a	
Haystack	217	10.0	219	9.2	2745	12.8	
Millstone			101	91.8a			
Goldstone 13			294	38.2ª	4	10.0	
Goldstone 13/14	9	9.2	14	7.6	300	11.4	
Goldstone 14	22	8.6	44	13.4	699	10.7	

^aThese data are mostly pre-1967 and are therefore of inferior quality. They were severely down-weighted in the solution for DE96.

Table 3. Mariner 9 range points

N	Julian date	σ, μs
77	2441272 - 2441361	0.25
81	2441389 - 2441540	0.29
487	2441541 - 2441555 2441577 - 2441602	0.78
159	2441556 - 2441575	2.50

Table 4. Initial conditions for DE96 at JED = 2440000.5

	X	Y	Z	×	Ý	ż
Mercury	39613212152212402	07992249271906621	00210472644821684	00065068177349635	02334496959482579	01243509127932027
Venus	.51314496534407696	.47569311696366166	.18195103289848097	01430395390425983	.01268998424173737	.09662179814461211
E-M barycenter	46560528150753017	82519998260935337	35783545693101644	.01499701589608436	00731148534216278	00317060837122165
Mars	.37962076027589121	1.35704574564607161	.61271727151903939	01300955944175007	.00409016370735938	.00222665371074985
Jupiter	-5.00319484519892979	1.82688075283407510	.90602247007006508	00293019707629372	00613064736625428	-,00255843388108078
Saturn	8.96742825251964510	2.64550399235593838	.70618378072282999	-,00191923452569552	+00489. 48077881151	.00210622175450133
Uranus	-18-27980664272406345	.551297886U3856273	.49985225679319031	00016608178020526	00377332452204415	00165064743411798
Neptune	-17.40588807261312355	-23.12846930183423506	-9.0361798968845189	.00254753898492655	00163212016561988	00073280091550904
Pluto	-30.54206443486996224	.72846598993191850	9.48536191616126237	.00016695687538702	00315129249824488	00104741357093554
Moon	-UG241507UZ1913819	.00109691667563689	.00051657959182900	-,00024037598607113	.00044399757611900	.07024714640572278

Table 5. Initial conditions for DE96 at JED = 2440400.5

	X	Υ	Z	×	Ý	ż
Mercury	.35579225802852818	09553558795038100	08771341781617990	.00370878085075952	.02484978478002579	.012926867083429?5
Venus	.60335039103368874	35590627403346386	19848792149401189	.01115692320275806	.01548890175644374	.006275152420442/6
E-M barycenter	.10369343609917386	92763405459100837	40233969418894512	.01683353411416213	.00155502989800035	.00067422930170243
Mars	13247798511420306	-1.52698491837590848	-,00555416413326086	.01446220287768023	.00007535316752929	00035412072748342
Jupiter	-5.39419589360612898	-,77099130630934636	19891034640604549	.00100563213273023	00653502483791158	00282810724754165
Saturn	7.94825659222305001	4.50716451784081462	1.51995150656620628	00315918649938017	.00436628069094962	.00194190840814613
Uranus	-18-28274953224057165	95838029508928623	16157384510723329	.00017136827054677	00376985232669572	00165419831467837
Neptune	-16.37169530238563224	-23.76164286400216318	-9.32162501287255551	.00262280859695046	00153293150139#15	00069406324451172
Pluto	-30.45234996429795698	53244506611606822	9.05948968049085383	.00028204868379326	00315213325233073	00108164477606805
Moon	00083570230400173	00198544033336024	00108326741801347	.00059875220638771	00017415339286388	00008847729948642

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Table 6. Initial conditions for DE96 at JED = 2440800.5

	X	Y	Z	X	Ý	Ż
Mercury	34728269561965779	25381986218864915	10025263498815027	.01164351062650545	0179689[116169349	01081911400871376
Venus	27292586304963801	-,61918740902651846	26178891568528682	.01860072818427010	00659050914891825	00+14466035444846
E-M barycenter	·63882263318717906	72346636913903414	31372001688906114	.01308702020817010	.00988058239124062	.00428446489160722
Mars	-1.04327.45071490276	1.14960956706884462	.55569172568390742	01028575666404713	00707637525666370	00297212078404506
Jupiter	2411 /916644777000	-3.14910678678419728	-1.24730472084339510	,00462631146681421	00505962643185183	00228365398826391
Saturn	6.45961075831318936	6.10113012489479580	2.24336224732550775	004252234340#2256	.00355926591661656	.00165556582938204
Uranus	-18-1+311868042883858	-2.46053133735052510	82172654002059815	.0005261924+645336	00373578223608604	00164431746628520
Neptune	-15.30833404391927475	-24.35427381300139223	-9.59118895464151705	.0026933026967969%	00142947750428933	00065342805740851
Pluto	~30.31628535964815481	-1.79242607536033235	8.62025335983048894	.00039861158425064	-,00314665447539976	00111423522166506
Moon	00160130309746393	.00195720791406307	.00096848254655875	00045840878,59171	00027669746488587	00017069695004875

Table 7. Radii and masses in DE96

Planet	Radius, km	Inverse mass
Sun	696000	1
Mercury	2440,12a	6023600
Venus	6052.06a	408523.5
E-M barycenter		328900.53
Mars	3397.51a,b	3098710
Jupiter	71350	1047.355
Saturn	60400	3498.5
Uranus	23800	22869
Neptune	22300	19314
Pluto	7200	3000000
Earth	6378.156	
Moon	1738	

^bMean equatorial radius of triaxial ellipsoid.

Table 8. Miscellaneous constants used in DE96

	DENUM	90.
	JDEPOC	2440400.5
	CLIGHT	299792.458
	EMRAT	81.3007
	BETA	1.
	GAMMA	1.
	JZSUN	ů.
٠	AU	149597871.41056
	RRAI	.09270
	RDE1	.04203
	RRAZ	.53056
•	RDE2	.22865
	ADAY1	.31050
	ADAY2	.20547
٠	ADAY3	27266
۰	DDAY1	53091
	DDAY2	-1.40953
*	DD4Y3	.68113
	DELK	-1.19073
•	DELN	.14768
	RAD1	2440.12173
	RADZ	6052.05827
•	RAD4	3397.51482
	RAD5	71350.
	RAD6	60400.
	HAD7	23800.
	RADR	22300.
	RAD9	7200.
	RADS	696000.
	RADM	1738.
	RE	6378.156

THESE PARAMETERS ARE FROM THE SOLUTION FOR DE96

Table 9. Covariance/correlation matrix for DE96

	σ	DMW1	DP1	091	EDw1	DAI	DEI	DMW2	DP2	DG2	EDW2						
DMw1	.99672865-002	1.000	.028	.010	+985	28		-				DAR	DE.S	DMMB	DPB	DOB	E(:+d
DP1	+12414716-001	.028										-, 38	8 .09	0.989	061	076	.97
DG1	.12185161-001	.010	.018								+058	01	.010	9 .093	.817		
EDWI	.20968352-002	.985	.032						.760	.553	.056	= . 064			.409		.09
DAI	.96878842-005	282	060									396			059		.13
DET	.30159148-003	004	.023	029	261						168	+635			.012		. 96
DMwa	.99846108-002	.995	.093	.064	005				.001		035	.013			.002		30
DP2	.11577049-001	045	.525	.760	+982				+037		.615	388			.023		05
062	-11424654-001	009	725	+553	008					.01A	.048	033			.869	.486	.98
EDW2	.11146406-003	+613	.058	.056	+611				.018	1.000		029			-,464	.871	.09
DA2	-17184404-004	388	012	064	396	168			+048	.002	1.000	245	018		.019		.01
DE 5	-10018625-003	.090	.019	.017	.095				033	020					.013	009	40
DMWB	.99972998-002	.989	.093	.128	.976	102		+093	026	035	018	045				003	
DPB	-11360189-001	061	.817		059	-,290		,998	.093	.010	.616	388			.053	.007	.12
998	+11439646-001	076	385	.849	073	006		.023	.869	464	.019	.013			1,000	.014	.98
LDwB	.17498452-003	.972	.093	+132	+964	301		055	.486	.871	018	009		.007		1.000	.05
AB	.26095554-004	415		071	424	.677	023	.981	.095	.012	.656	405	.121	.984	054	.011	1.00
830	.15506337-004	.240	.056	+045	+242	242	.013	415	019	056	241	.938	152	416		011	43
M#4	.10029225-001	.994	.078	.097	.980	296	.071	.244	.050	023	.056	331	.162	.245		009	.22
)P4	-11491115-001	.070		921	.067	.012	006	.999	.057	.004	.615	397	.092	.990		016	.98
964	-11313089-001	.016		184		009	-,001	.003	903	419	009	+018	007	064			06
D#4	.96633976-003	.991	.077	.099		333	.004	.071	+403	899		015	+013	.05A		596	.05
Au	.41780337-004	415	021		426	.673	006	.996	.057	.005		449	.100	.997		015	.98
E4	.25960616-004	086			110	.126				024	264	.932	162	418			441
P5	.18051642-001	.143	.005	+017	+143	177	005	089		012	304	.177	190	091			254
95	-18453470-001	.005	.071	083	.005	.003	001		.003	*00%	.074	238	+015			001	.135
DW5	.18152235-001	009	.110	+057	-+008	.002	.000	.003		115	.004	.004	.000	000		113	.000
A5	.46853976-002		026			059	007			061	.004	.003	.003	.007	.135	.004	.008
£5	.88020396-003	021	.017		024	.234		019	.016	+005		079	003	195 .	.020		186
MBG	.51144925-002	.156	.024	.017	+155		007	.158		013	.007	.316	027	016			017
P6	.27230875-001	045 -		005 -	041				018	006	.135	.067	.031	.158		003	.177
96	.19778160-001 .19989124-001	003	.070	.023 .	+003	000	.000	.004		048		153	.020		.022	.003 .	041
Dw6	.99018434-002	003 -			\$002	.002		002	.020		000	000	.001	.005	. ORO .	011	.005
A6	.30765173-002	.008	.002	*001	*00B	.004	.000	.008		001	.005	.003	001		.007	.055	.001
E6	.10842283-001	.103	.033	.011	.097	.166	.005	.106		017	.070		000	* 00A	.002 -	.000	.008
Mw 7	.80256468-001					000	.000		.000			.222	028	.107	.034 -	.005	.097
27	.23300299-001					034	-,001		.006			000		000 -			.000
7	.19253424-001	000	.016		000	.001	.000	.002	.032	.002	.002	.046			.007		.019
w7	.35253706-001					.001	000 .	003 -	.032				.000		.027	.017	.004
17	.28928165-001			.000	.002	.003	.000	.002		.000	.001				.043		.004
.7	-14419578-001			.004	+033	.050	.001	.036		.005	.024		001	.002	.000	.000	.001
w8	.39976929+000				.005	.009	.000	.005		.001	.003		001	.036		.001	.033
8	.24731825-001							.006 -	.001			.011					.005
18	.23479725-001					.003	.000	.012	.087 -	.067		.003	.001				.005
w6	.17867066+000				+004			.008 -	.039		.005						.014
8	.26803061+000						.000		.000	.00n		.000	.000				.009
8	.15079304+000				.007	.011	.000	+008		.001	.005		.002				.000
	.30555712-001					.001	.000	.000		.000			.002				.007
A 1	.39782975+000							.407	.021	.027		.927	.156				.000
£1	.45770535+000				.001	*00C -	.000 -	.001 -	.002	.001 -							. 440
A2				.006 -	.002	.001	.000 -	.001		.003 -						.000 -	.001
E2	.26998911-001		017 -	002 -	020 -	.001 -							.000 -	.001	005	.006 =	.001
AYI	.26763081-001		.003			.000 -							+003 =	.022	017	.007 -	.021
AY2	.27617463+000					.001 -						.002	.006 -	.002			002
AY3	.94226642-001					.000 -											040
AYI	.29005544+000					.000 -						.001					001
172	.27341903+000		011 .											.003 .			003
173	.96094159-001	034	040 .										.000 -				004
, K	.28776840+000	.007	013														000
	.55246045-001												.000				005
,N	.47872361-001	.087															212
2	-11297766+000																053
4	.71473749-001	.029	002 .														004
	.44120139-001	.247									060	130 -	.289				

Table 9. (contd)

	σ	DAB	DEB	DMMA	DP4	094	E0#4	DAH	DE4	DMWS	DPS	595	EDWS	DAS	Des	DMM	
DMw1	.99672865-002	415	.240	.994	.070	.016	,991	415	5 =.086						DES	Dw#6	DP6
OP1	.12414716-001	013	.056	.078									19		.156	045	00
D@1	.12185161-001	071	.045	+097	-,921	184							026	.017	.024	020	.07
Dwl	+20968352-002	424	.242	.980	.067	.016							020	004	.017	005	.02
PAI	.96878842-005	.677	242	296	.012							008	186	024	.155		00
E1	.30159148-003	.013	.071	006		009					7 .003	.002	059			119	00
MWZ	.99846108-002	415	.244	.999	001	.004					5 001	.000	007		007		7.5
P2	.11577049-001	019	.050	.057	+003	.071	, 996			.14	4 .006	.003			.158		.00
992	.11424654-001	026	023		-,903	.403			*.033	.00	3 026				.019		.00
Dw2	.11146406-003	241	.056	+004	419	899	.005	024	012	.00			.005		006		.06
A2	.17184404-004	.938		+615	009	.031	.611			.074						.011	04
E.2	.10018625-003	152	331	397	+018	015	-,449		.177	234					.135	032	.00
HWE	.99972998-002		.162		007	.013	.100								.067	153	00
PB	-11360189-001	416	.245		064	.058	.997	418	091	.143					.031	.050	.00
98	+11439646-001	.018	.050		603	.793	.022	.002	019				020	018		047	.00
D₩B	17498452-003	011			804	596	015		023					.024		055	.08
AB	.26095554-004	434	.229		n65	.052	.981	449					.005		003	.003	01
EB			-,358	-,425	.021	016	-,481	.994	.190			.008	186	017		041	.00
Mw9	15506337-004		1.000	.248	032	.052	.273	341	060			.003	084	.338	.071	164	00
P4	.10029225-001	425	.248	1.000	028	.048	,998	426	089			.009	061	038	.065	.037	.00
94	.11491115-001				1.000	.002	029	.028	.030	.145		.003	104	022	.157	*.045	.00
044	-11313089-001	016	.052	.048	+002	1.000	.048	019				083	.015		004	.010	03
A4	.96633976-003	481	.273	-	029		1.000		.013	.006		.106	017	.010	.012	015	.07
E 4	-41780337-004	.994	341	426	+028		481	1.000	0.6	.160		.003	180	046	.144	032	.00
Hu5	.25960616-004	.190	.060	089	.030		096	.242	.242	249			076	.333	.074	163	00
P5	.18051642-001	255	.002		-+004	.006	.160	249	1.000	082	.005	007	116	.073			00
	.18453470-001	.005	005	.001	+071	.094	.001	.005	082	1.000	.001	005	.762	727	551		00
95 0 w 5	.18152235-001	.003	.009	.003	083	.106	.003		.005	+001	1.000	.063	006	001	002	002	.001
	.46853976-002	084 -	061	194			180		007	005			009	.002		003	.01
15	.88020396-003	.338 -			+006		046		116	.762			1.000	663	166		002
	.51144925-002	.071	.065		004	.012	.144	.333	.075	727		.002	663	1.000		170	.001
10	.27230875-001	164		.045					089	-,551	002	.008	166	.538		083	.001
*	.19778160-001	000	.005	.003	039	.071			034	.099	002	003	.111				.012
tu	·19989124-001	.003 -	.002				003			200	.009	.015	002	.001			.000
W6	.99018434-002	.006 -			.001	.002				002	010	004	002	.003			.066
16	.30765173-002		.050		+016	.024	.008	.005	.001	004	.000	.000	006	.006	.005		.002
.6	.10842283-001	000		.000			.083	.236	.049	140	.003	.005	170	.250		.847	.007
(w 7	.80256468-001	049		.020				000	.000	*00g	000	000	.000		.000	.426	.001
7	.23300299-001	.001	.001						010	.030	001	001			.027		.001
7	.19253424-001			.003	.030	.011	.003		001	001	.003	.007	002	.002		.002	
) w 7	.35253706-001		.001					001	.000	.001	006	009			.002		.006
7	.28928165-001		.014		.000	.000	.001	.005	.001	003	000		003	.005			.007
7	-14419578-001		.003		+005	.007	.029	.07:	.015	043	.001		054	.078			.000
w6	.39976929+000	011			.001	.001	.004	.012	.002	00A	.000		009		.040 -	+054	.001
6	.24731825-001						.005 .	011 -	002			000				.009	.000
3	.23479725-001				.052	.099	.011 -		003	.001	.030		.004		.007		.000
w 6	.17867066+000				.019 -	.053 -	.008	. 1	.001			020					.031
9	.26803061+000					.000	.000 -	.000 -	.000	.000		000					.019
3	.15079304+000				.001	.002	.007	.015		009	.000						.000
				.000 -	.000	.000	.000	.001		001	.000		.012			.012	.000
1	.30555712-001			.417 -	.026	.017	.471 -		.281				+001			.001	.000
	.39782975+000	000 -	.000 -	.001	.001 -	.002 -		.000				003				.161	.000
1	.45770535+000	.000	000 -	001 -	008							.000	.000	.000 -	.000 -	.000 -	.000
2	-26998911-001									.000 -	001	.001	.000	.00n	000 -		000
2	.26763081-001						.021 -		.000	.001 -	002 -	.003	.00B -	.007			
Y 1	.27617463+000							.000 -	.001	.000 -		7.55					002
Y 2	.94226642-001					014 -		.002 -	.000								001
Y 3	.29005546+000					006 -											002
Y 1	.27341903+000			003		001 -	.003 -										001
42	.96094159-001				007												000
Y3				010												001 .	002
K	.28776840+000	.000									.028					001 .	004
	.55246045-001	.403	072 .		7 7 7							.003 -	002				002
N	.47872361-001	.196										.043 -	321				035
1	.11297766+000																082
2	.71473749-001											.001			IIII		000
4	.44120139-001														22.2		
				F 20 -+	043 .	015 .	293	663	113 .	.269 -	.007					012	000

Table 9. (contd)

Day 1	σ	D96	ED#6	DA6	0E.6	DWR.	7 097	097	EDW	7 DA7	D€7	DWM9	DPB	Dos	EDWA	DAR	DEB
DMW1 DP1	.99672865-002	003			00			000	0 .00	2 .0	35 .00	500	6 .00	3004	.000	.008	0.0
091	.12414716-001 .12185161-001	026				000				0.0	10 .00						
EDw1	.20968352-002	+043				000		600									
DAI	.96878842-005	002			00			000				500					.000
DEI	.30159148-003	002			00			100		3 .01		900				.011	
DMw2	.99846108-002	002		.005				000			1 .00	000				.000	.001
DP2	.11577049-001	.020		.030	00						6 .00	5 00			.000	.008	.000
De2	.11424654-001	.052		017				203					1 .087			.002	.000
EDw2	.11146406-003	000	.005		=+000									.039		001	000
DA2	.17184404-004	.003	.005	.222				200				3 004		005	.000	.005	.000
DE2	-10018625-003	001	000	028	+000			100				101		.001	000	.014	.001
DMWB	.99972998-002	.001	.008	.107				00							.000	002	000
DPB	.11360189-001	007	. 202	.034	000			7 04						009	.000	.008	.000
Des	.11439646-001	.055	000	005	000						17				000	.002	.000
EDWB	.17498452-003	.001	.008	.097	030										.000	000	000
DAB	.26095554-004	.003	.006	.238	000			001			10	005			.000	.007	.000
DMW4	.15506337-004	002	000	050	.000	.01		002						.001	000	.015	.001
DP4	.10029225-001	.000	.008	.103	000			003							.000	003	000
De4	.11491115-001 .11313089-001		001	016	.000	.003	030	+017						008	.000	*00#	.000
EDW4	.96633976-003	039	.002	+024	000			041					.099	.019	.000	001	000
DAY	.41780337-004	000	.008		000			003		.02			.011	053		.002	.000
DEN	.25960616-004	003	.005	.236	000			001	.008	.07					000	.007	.000
DMW5	.18051642-001		004	-049	+000			.000					003		000	.015	.001
DP5	.18453470-001	010	.000	140	+000	.030		.001		04	3006		.001	.001	.000	003	001
DQ5	+18152235-001	004	.000			001							.030	019		.000	.000
EDW5	.46853976-002			- 170	*000	001							.035	020		.000	.000
DAS	.88020396-003	.003	.000	-	000	054		.002					004	.004			001
D£5	.51144925-002	.001	.005		000	027		002					*005	003	000	.C17	.001
DNW6	.27230875-001	008	.040	847	+426	937		.002					.003		000	.009	.000
096	.19778160-001		002	+007	.001	001	.006	007					005	.004	.000	012	001
EDW6	.19989124-001	1.000	.005	.006	004	001		.003	.000	.001		- 9 - 0	.031		000	.000	.000
DAG	.99018434-002 .30765173-002		1.000		074	002	.000	000	.000	.003	.000		017	.011	.000		000
DE6	.10842283-001	.006			357	055	.003	003	.005	.080		013	.001		000	.001	.000
DMW7	.80256468-001				1.000	+000	000	.000	000	000		.000	000	006	000	.017	.001
DP7	.23300299-001	001		.055		1.000	001	.003	.945		.135	.003	001	.001			000
097	.19253424-001		.000		000	001		013	000	.001	.003	000	.019		.000		000
EDW7	.35253706-001	.000	.000	.003	.000	.003	013	1.000	.002	003	.002	.000	020	.012		.000 -	.000
DA7	.28928165-001	.001	.003		.000	.945		.005	1.000		.031	000	000		.000	.000	.000
D€7	-14419578-001	.000	.000		- 000	972 .135	.001	003	-,948	1.000	.006	004	.001		.000	.005	.000
DMW8	.39976929+000			.013	.000	.003	.003	.002	.031	.006		001	000		.000	.001	.000
OP8	.24731825-001	017	.001			001	.019		000	004	001	1.000	.002			.985	.925
998	.23479725-001	.011 -		.006	+000	.001	012	0.0	000	.001	000		1.000	411 -		.001	.000
EDW8	.17867066+000	.000 -	.000 -	.000	.000	.000	000	.000	000	001	.000			1.000	.006	.006 -	.005
DE 8	.26803061+000	.000	.001	.017 -		004		000	.000	000	000		002		.000	.985 -	.932
NU N	·15079304+00G		.000			000		000	.000	.005	.001		001	.006		.000 -	.975
RAI	.30555712-001	003 -		.234	.000		001	.001	005	070	012	.925				.975 1	.000
	.39782975+000	.000 -	.000	.000	.000	000	000	.000	.000	.000	.000	.011		001		.015 -	.001
DE1	.45770535+000	.000 -	.000 -	.000 -	.000	.000							000			.000	.000
RAZ	.26998911-001	.001 -	.000 -		.000		001		- T. C. T.	000		.000	.000 -	000 -	.000 -	.000	.000
DE2	.26763081-001			227	.000	.000			000	003			004	.003	.000 -	.001 -	.000
DAYI	.27617463+000				.000		001	000	000		000	.000		.001 -			.000
DAYZ	.9422664Z-001	000				.000		.001		007	001		.005				.000
DAY3	.29005544+000		.000 -	222	.000	.000		000		.000	.000	000			.000		.000
DAY1	.27341903+000				.000	.000				001	000	.000					00
DAY3	.96094159-001		.000 -	003 -	.000	.001					000	.000				.000 -	.000
ELK	.28776840+000			003			001	.001	.000	.001	000	.000					.000
ELN	.55246045-001			478 -		.102		.025	.008	.149			.005				.000
A01	.47872361-001 .11297766+000			198			051	.052	.004	.065		024 011 -					.002
AD2	.71473749-001				000	.000	.000				000	.000	.298				.000
AD4	.44120139-001			018	000	.004 -	000			0.000	001						.000
-		-,001	004	178	000	.037 .	000		.003 -		009			.000			000

Table 9. (contd)

	J	AU	RRAI	RDE1	RRA2	ROES	ADAY	1 ADAY	2 4044	3 004		S DDAY					
DMW1	.99672865-002	.406	600	1 00	2 02							S DDAY	3 DELK	DELN	RADI	RAD2	RAD4
DP1	-12414716-001		00		2 01		404 501						7 .21	2 .087	.002	.029	.24
EDW1	.12185161-001	.072					501							7215	.206		
DAI	.20968352-002	.417			02										034	+007	
DEI	.96878842-005 .30159148-003	668		*001	00				000		603				.073	+031	
DMW2	.99846108=002	025			000	00			=.000	.00					027		45
DP2	.11577049-001	.407			-+05	00		001	00	00					399		
092	-11424654-001	.027			011			+008	.002	.01					.003		
ED#2	.11146406-003	.262					026		000	00	8 .20				002		
DA2 DE2	.17184404-004	927	000				002		002				.149			060	.03
DMy8	*10018625~003	.156		-+000	003		002		.000		000					130	63
DPB	.99972998-002 .11360189-001	.408		001	022	002	042				0000				.001		.06
Ues	.11439646-001	011			017		017	.009							.003	.029	.25
EDWB	-17498452-003	.013	000					.002	.001						004	003	004
DAB	.26095554-004		000		021		040						.212		.063	.004	.032
DEB	.15506337-004				002		002	.000			002	.000	.403		004	.039	.254
DMW4 DP4	.10029225-001		001	001	021		006		000	.000	.001	000	072			077	674
Des	.11491115-001	026	.001	008	+005		.011		003		010	+005	.220		.003	.030	.256
EDW4	.11313089-001 .96633976-003	.017	002		018	.000		.006	.002		272	+008	082		000	003	043
DA4	.41780337-004		001		021		040		003	004	100		.129			003	.015
DE4	.25960616-004	281	.000	000	002		002		000		004	.000	.185		.003	.034	.293
DMW5 DP5	.18051642-001		000	000	+000		000	.001	000	000			.082				663
005	.18453470-001	005			002	.000	001	000	.000	.001			253		001	064	113
EDWS	.18152235=001	003	000		003		003	.000	.000	.001			.035	078			007
DAS	.46853976-002 .86020396-003	.071	.000	.000	+008	.001	.018	.000	.000	.002			.043	091	001	001	-003
065	.51144925-002	320	000		007	001	016	.000		002		002	321	-,134	002	.004	.173
DMM6	.27230875-001		000	000	007	001	014			002		.002	.460	.204			301
0P6	-19778160-001		000		+006	.000		000	.001	.001	.001	=.002		132	.001		076
Dw6	-19989124-001	003	.000	.000	.001	.000	002	001	.000	.002	.004	002				.012	.121
A6	.99018434-002 .30765173-002	005	000	000		000	001			001	.014		013	.049	.000		001
Eo	*10842283=001	-,234		000			021			000		.000	.017	.005 -			004
Mw7	.80256468-001	.000	000	000		000	.000			000		.003	.478	.198 -			17A
P7	.23300299-001		000	.000	*002	.000		000	.000	.000	.001		102	044		.000	.000
97 Dw7	-19253424-001	.001		000	+001		001	.001	.000	.001	.008	001		051	.000 -	.004	.037
A7	.35253706-001	005			000			001	000			.001	025	.052	.000	.000	000
£.7	.28928165-001 .14419578-001	070	.000	000	003		007		000	000	000	.000	.008	.004 -	.000 -		.003
MWB	.39976929+000	012	.000	000 .			001			001		.001	.149	.065 =	.001 -		.053
P8	.24731825-001	.004	000	.000	.000	.000	.001	000	.000	.000		.000	-025				.009
98	.23479725-001	001		.000	004	.002	005	.002	.001	.004		005		011	.000	.001	.009
A8	.17867066+000			000	.000				.000	003			.078		.000 -	.001	.002
EB	.26803061+000	015					.000 -		.000		.000		.000				.000
U .	.15079304+000 .30555712-001	001	.000		.000		000	.000 -	.000			.000	+032		*		.011
RAI	.39782975+000	1.000		.000	.002 -	000		.000	.000	.000	.000	.000	.005				.001
DE1	.45770535+000			.000		.000	.000 -	.000 -	7.2		000		.000				.708
SAF	.26998911-001	.002			+000	.000		.000	.000	.00n		000					.000
E2	-26763081-001		.000 -			.000		+014	.059 -	.000	.001		.022				.000
AYI	-27617463+000					.000 -		.000		.040	.025	.037					.002
SYA SYA	.94226642-001	000 -	.000		.014	.000 -			.994			000 -	.044 -				.000
AYL	-29005544+000	.000 -	.000		.059	.000 -		.000	.026	.000		.000	.003 -	.007			.004
SYA	.27341903+000 .96094159-001	.000 -	.000	.000 -			.000		.000	.000		000 -	.003 -	.003			.000
AY3	.28776840+000	.003 -		.002	-001	.025 -	.001					.996	.000 -		000		.000
LK	-55246045-001					.037 -	.000 -		.000 -		.022	.022	.005 -				.000
LN	.47872361-001						.044	.003 -	.003	.000		.000 -	.000				.000
D1	.11297766+000							.007 -	.003 -		.030		.074 1	.074			304
D2 D4	.71473749-001 .44120139-001					.002				.000 -	.000	.000 -	.003 -	000 1			016
				.000		* UUE	+ MIII =	.000	.000	00-	.000 -					U103 6	177 \$ 65

Table 10. The masses and Keplerian elements of the asteroids used in the force model

Asteroid	$m = [m_{\odot}^{-1}]$	a	е	f, *	Ω, •	ω, *	M, * (2440000.5)	n, ″∕day
Ceres	5.9×10^{-10}	2.7663	.078796	10.604	80.420	69.762	58.455	771.167
Pallas	1.3×10^{-10}	2.7687	.241216	34.848	172.802	310.110	46.750	770.201
Vesta	1.2×10^{-10}	2.3619	.088744	7.137	103.631	149.932	85.095	977.467

^aThe masses of Ceres and Pallas are from Schubart (Ref. 12), the mass of Vesta is from Hertz (Ref. 13), and the elements are from the Astronomical Papers (Ref. 14).

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